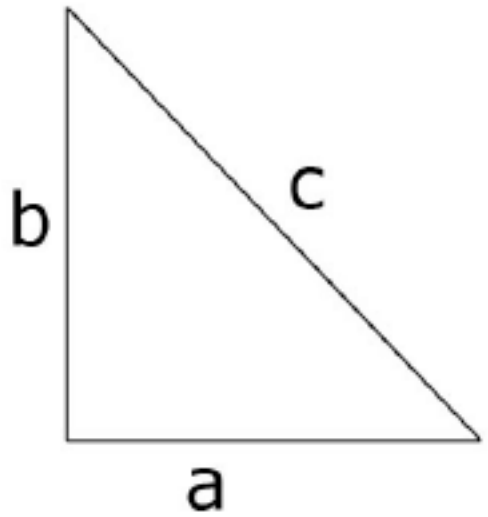
Right-angled triangles

Pythagoras' Theorem
You need to be able to calculate missing sides in right angled triangles, given two sides by using Pythagoras' theorem.

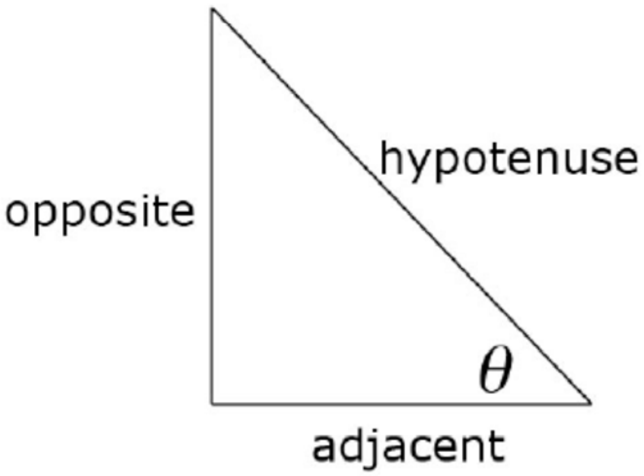


$$a^2 + b^2 = c^2$$

$$c^2 - b^2 = a^2$$

$$c^2 - a^2 = b^2$$

SOHCAHTOA
You need to find missing angles and missing sides in right angled triangles by use SOHCAHTOA (**S**in **O**pposite **H**ypotenuse **C**osine **A**djacent **H**ypotenuse **T**angent **O**pposite **A**djacent)



$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{o}{h}$$

$$\cos\theta = \frac{adjacent}{hypotenuse} = \frac{a}{h}$$

$$\tan\theta = \frac{opposite}{adjacent} = \frac{o}{a}$$

Angles from 0° to 360°

The calculator will give you one answer, usually between 0° and 90° for an equation in the form sin $\theta = x$. However there will be two answers between 0° and 360°.

A basic rule is this:

- Calculate one value using your calculator

  sin θ = 0.5
  Type in $sin^{-1}(0.5)$ = 30°

  cos θ = 0.7
  Type in $cos^{-1}(0.7)$ = 46°

  tan θ = 1.2
  Type in $tan^{-1}(1.2)$ = 30°

- For sin then the second answer will be 180 - θ

  180 – 30 = 150
  Answers are 30° and 150°

- For cos then the second answer will be 360 - θ

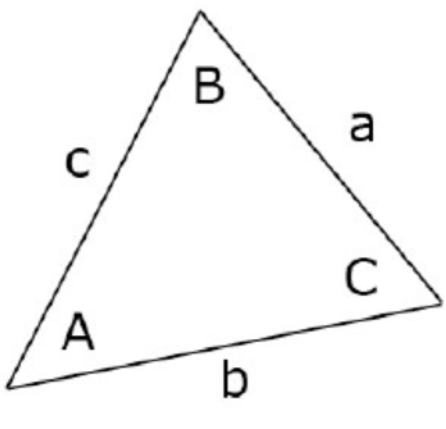
  360 – 46 = 316
  Answers are 46° and 316°

- For tan then the second answer will be 180 + θ

  180 + 50 = 230
  Answers are 50° and 230°

Non-right angled triangles

Formulae for non-right angled triangles are based on the diagram and notation below below.



You should be able to apply the sine rule, cosine rule, and the area of a triangle. All of these formulae are in the IB formula booklet.

The sine rule (non-right angled triangle)

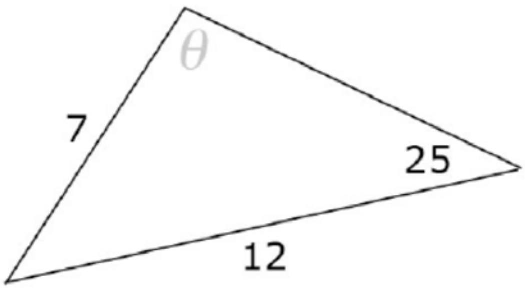
The sine rule is used when:

- You are given two sides and an angle you are missing an angle.
- You are given two angles and a side and you are missing a side.

Missing side:-	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Missing angle:-	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Example

The missing angle in the triangle below is known to be obtuse. Find the missing angle.



Use the formula $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$, where a = 12, b = 7 and B = 25º.

The solution will be: $\frac{\sin A}{12} = \frac{\sin 25}{7}$

$\sin A = \frac{\sin 25 \times 12}{7}$

$\sin A =$ 0.724

$A = \sin^{-1}(0.724)$

A = 46º

But as A is obtuse (between 90º and 180º) the answer will be

180 – 46 = 134º.

The cosine rule (non-right angled triangle)

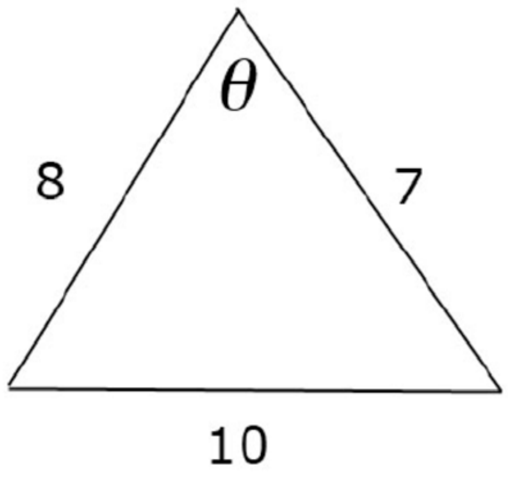
The cosine rule is used when:

- You are given three sides and you are missing an angle.
- You are given two sides and the angle opposite the missing side.

Missing side:-	$a^2 = b^2 + c^2 - (2bc\cos A)$
Missing angle:-	$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Example

Find the missing angle in the triangle below.



Use the formula $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$, where a = 10 as it is opposite θ, b = 8 and c = 7.

The solution will be: $\cos\theta = \dfrac{7^2 + 8^2 - 10^2}{2 \times 7 \times 8}$

$$\cos\theta = \frac{13}{112}$$

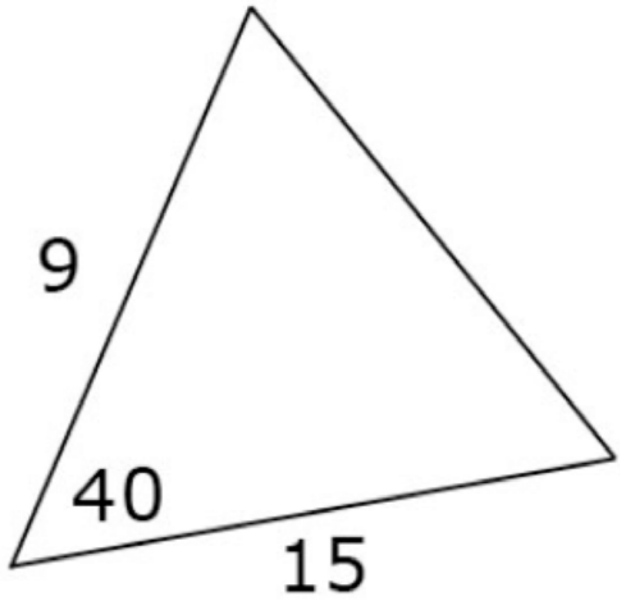
$$\theta = \cos^{-1}\left(\frac{13}{112}\right)$$

$$\theta = 83^{\circ}$$

Area of a triangle (non-right angled triangle)
The area of a triangle can be found if two sides are known and the angle between them is give. The formula is given below.

Area of a triangle:-	$\frac{1}{2}ab\sin C$

Example
Find the area of the triangle below.



Use the formula $\frac{1}{2}ab\sin C$, where a = 9, b = 15 and C = 40°.

$\frac{1}{2}$x15x9xsin(40)

= 43.4 units^2

Bearings

Bearings are always 3 figure angles. The basic rules for finding a bearing are:

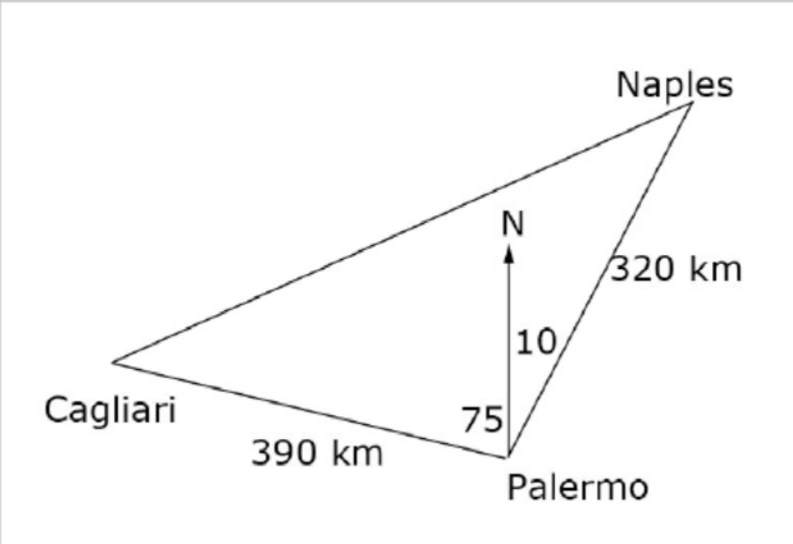
- Locate where you 'are'. This will be the location that follows *from*. E.g. If you are asked to find the bearing of Bangkok from Columbo, you will be at Columbo.
- Draw a line between the two locations.
- Draw a line going north from where you are.
- Draw an arc starting at the line going north and stopping when you reach the destination line.
- The angle of the arc is the bearing you need. Always give 3 figures in your final answer, so if the angle measure 35º, the bearing will be 035º.

Guided example

Two ships set sail from the port of Palermo. One sails to Cagliari a distance of 390 km on a bearing of 285º. The other ship sails to Naples on a bearing of 010º and a distance of 320 km.

(a) Draw a diagram to show the information given above.
(b) Use you diagram to find the area in km^2 between the towns of Palermo, Cagliari, and Naples.
(c) Find the bearing and distance from Cagliari to Naples.

Answer (a)

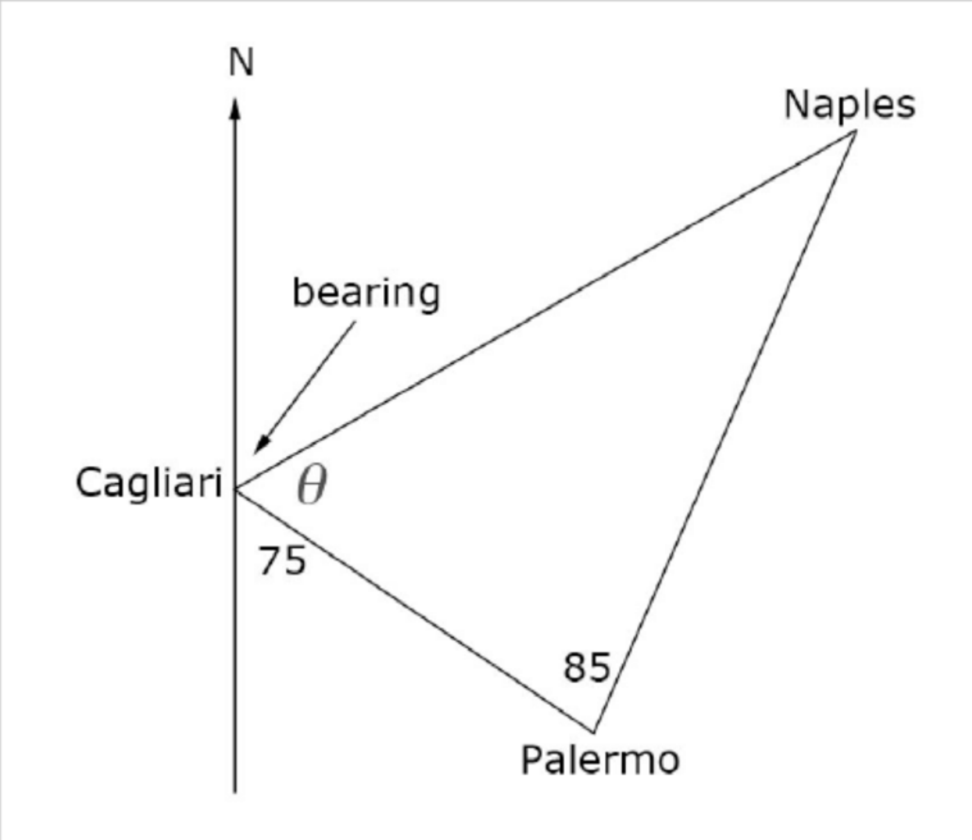


Answer (b)

As the triangle is not a right-angled triangle then the $\frac{1}{2}abSinC$ formula:

$\frac{1}{2}\times 390\times 320\times \sin(85)$ = 62163 km^2

Answer (c)
The direct distance between C and N can be calculated using the cosine rule:
$CN^2 = 320^2 + 390^2 - (2x320x390xcos85)$
$= 232745$
CN = 482 km



The bearing is slightly harder. Look at the diagram below and the bearing is marked. The angle 75º is calculated using the fact that the two lines north (from P and from C) are parallel and therefore alternate angles equal.
Calculating θ will adding this with 75, before taking it away from 180 will give us the bearing.
The angle θ can be calculated using the sine rule:

$$\frac{\sin\theta}{320} = \frac{\sin 85}{482}$$

$$\sin\theta = 0.661$$

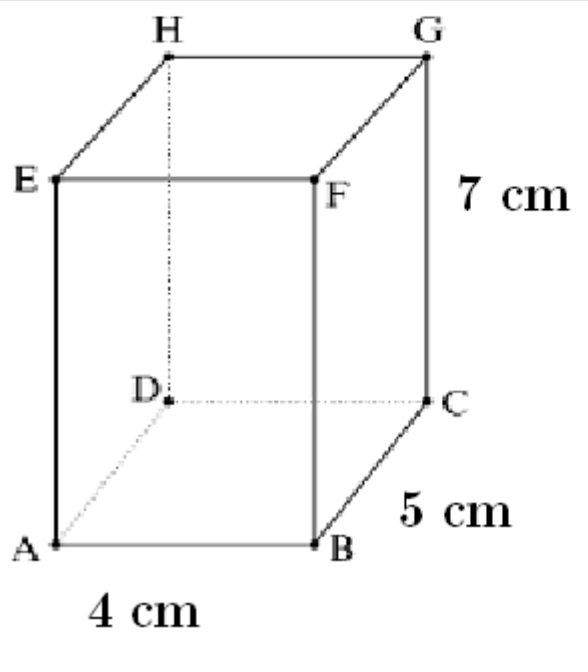
$$\theta = 41$$

The bearing is therefore 180 – (75 +41) = 064º

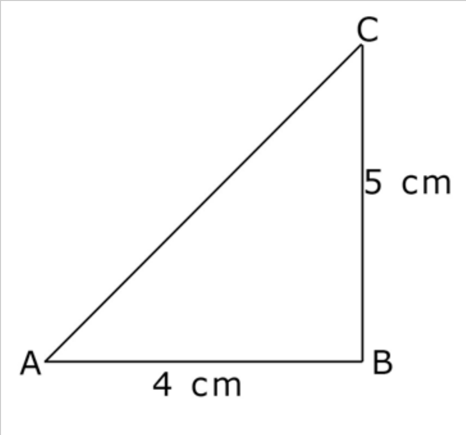
3-d trigonometry
3-d trigonometry basically involves breaking down the 3-d shape into 2-d right angle triangles and using either Pythagoras' theorem or SOHCAHTOA.
Examples of this can be seen in the worked example below.

Guided example
In the diagram below *AB* = 4 cm, *BC* = 5 cm, and *CG* = 7 cm.

(a) Calculate the length of *AC*.
(b) Calculate the length *AG*.
(c) Find the angle made between the base of the cuboid *ABCD* and the line *AG*.

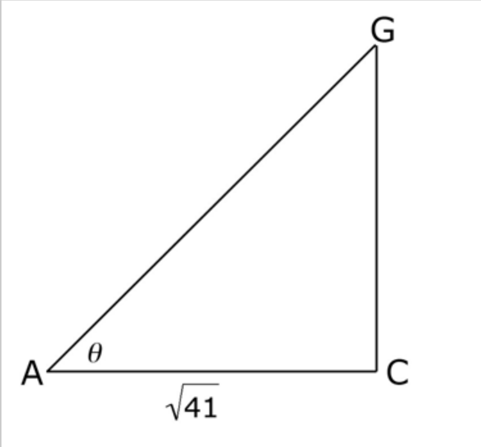


Answer (a)



By drawing the right angled triangle ABC we can see that a simple use of Pythaoras' theorem will result in AC = $\sqrt{4^2+5^2}$ = 6.4

Answer (b)



Again by drawing a right angled triangle ACG we can see that AG = $\sqrt{6.4^2+7^2}$ = 9.49

Answer (c)
Using the same right-angled triangle you need to calculate the value of θ by using tan.

tan θ = $\frac{7}{\sqrt{26}}$ = 54°