Set Theory

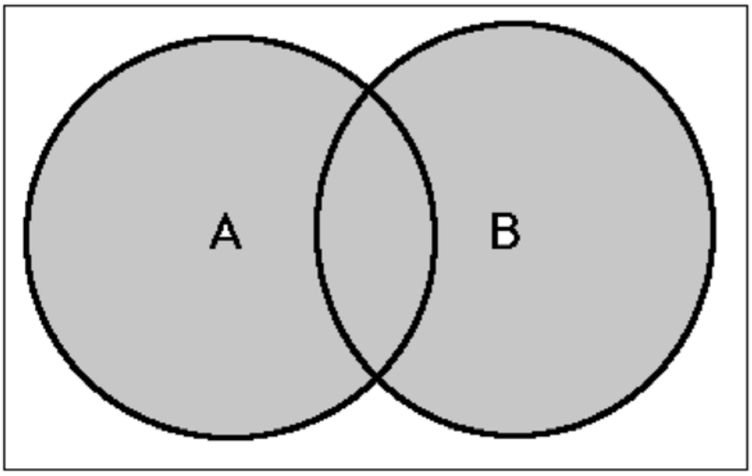
Set notation

Symbol	Definition	Means…
U	Universal set	All the elements given in a question
$\in$	Is an element or member of	Number/object is in the set
$\notin$	Is not an element of	Number/object is not in the set
‘	Complementary set	Opposite of the set or ‘not the set’
$\varnothing$	Null	An empty set with no members
$\cap$	Intersection	When an element is in two (or more sets at the same time)
$\cup$	Union	All the objects of two or more sets.
$\subset$	Setset	A smaller set contained wholly within a larger set
	n(X)	The number of elements in a set.

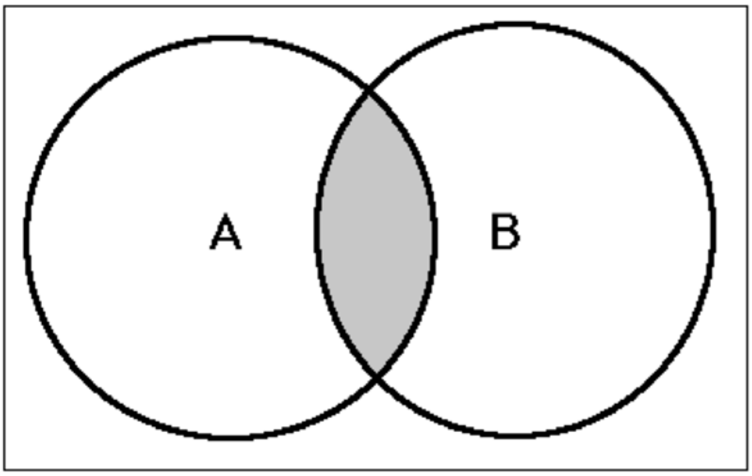
Venn diagrams

Venn diagrams are a visual way of representing the set notation. Below are some Venn diagrams to show the set notation symbols above.

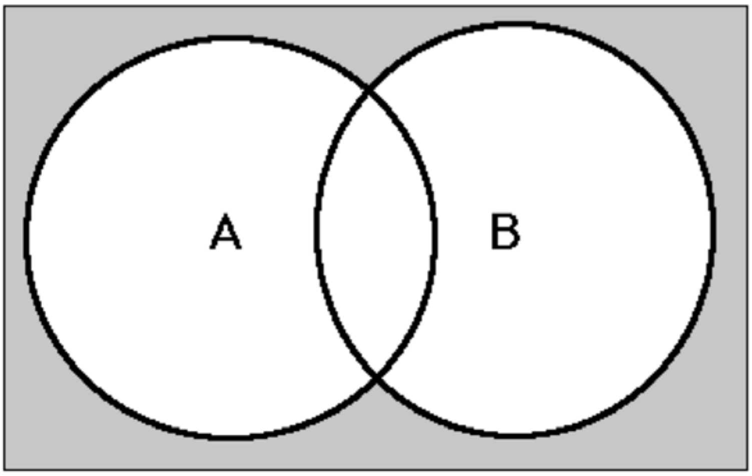
$A \cup B$



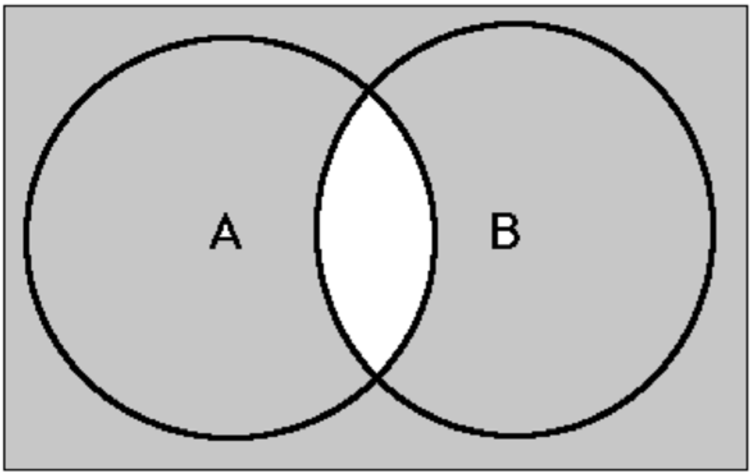
$A \cap B$



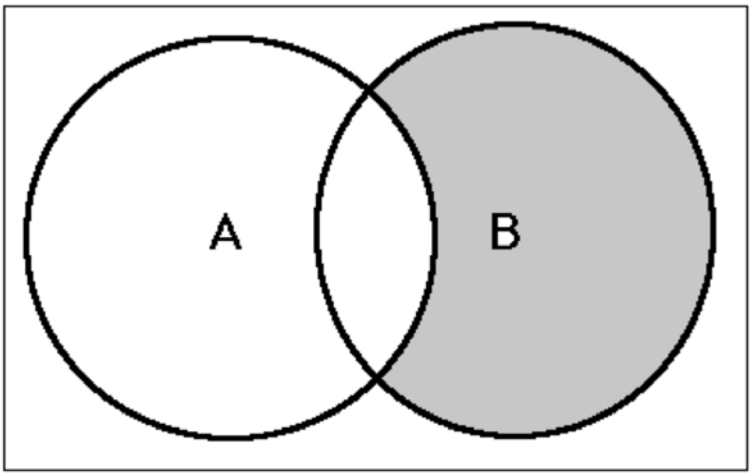
$(A \cup B)'$



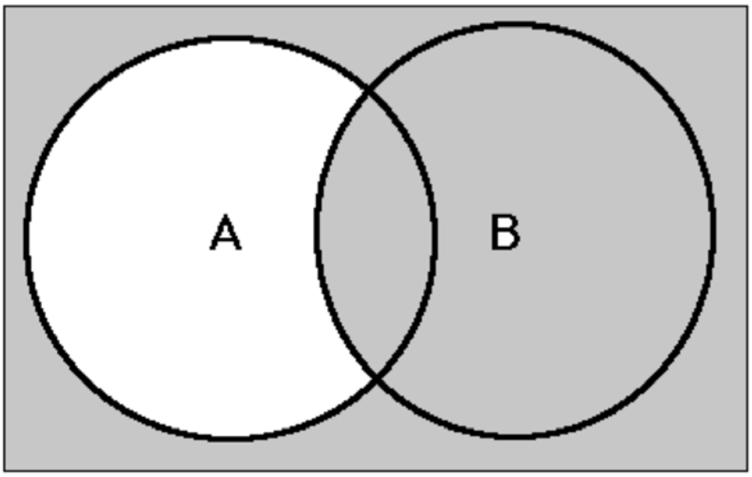
$(A \cap B)'$



A’ ∩ B



A’ ∪ B



Guided example

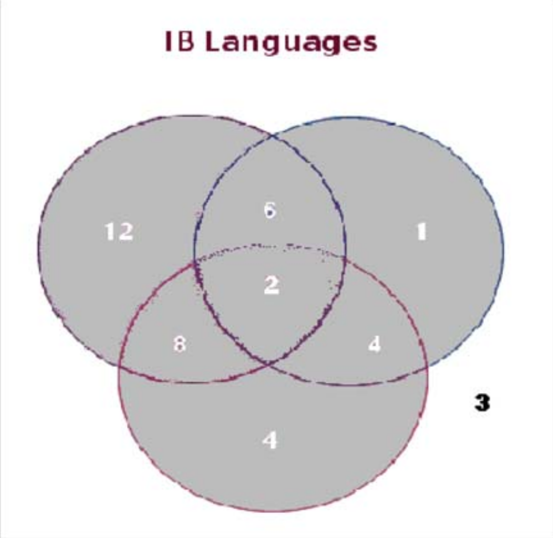
A group of 40 IB students were surveyed about the languages they have chosen at IB: E = English, F = French, S = Spanish.

3 students did not study any of the languages above.
2 students study all three languages
8 study English and French
10 study English and Spanish
6 study French and Spanish
13 students study French
28 students study English

(a) Draw a Venn diagram to illustrate the data above. On your diagram write the number in each set.
(b) How many students study only Spanish?
(c) On your diagram shade $(E \cup F)'$, the students who do not study English or French.

Answer (a)

- There are 3 sets within a universal set. So the Venn diagram will have 3 intersecting sets. We know that the sets must intersect as there are some students who belong to all 3 sets. Draw a Venn diagram with 3 intersecting sets.
- Now we can work carefully through the statements and fill in as we go along.
- The 3 students who do not study any language go inside the box, but outside of the circles.
- 2 students must go in the intersection of all three circles.
- 8 must go in the intersection of E and F, but remember that 2 have already been filled in the intersection of all 3 sets. So only 6 will be put in the intersection.
- Likewise the English and Spanish intersection will have 10 – 2 = 8 and only 4 will go in the French and Spanish intersection.
- In the French set there should be 13, but thus far we have filled in 6+2+4 = 12. So only 1 student is left to go in the French only set.
- Likewise 28 students study English, but we have filled in 16 of these students, so 12 go in the English only section.
- There are 40 students altogether in the survey. Counting all the numbers filled in thus far reveals we have put 36 numbers on the diagram. It is important to remember the 3 outside the circles, but in the box. Fill in 4 in the Spanish only section.
- The final diagram will look like this:

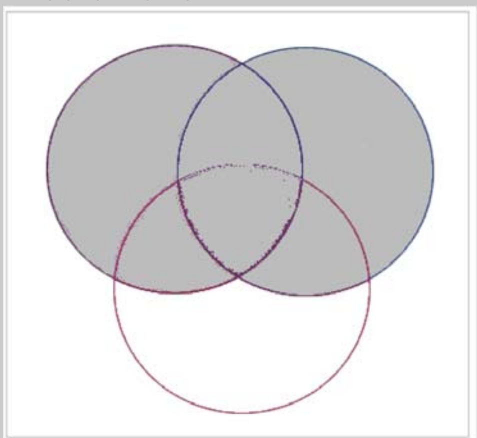


Answer (b)

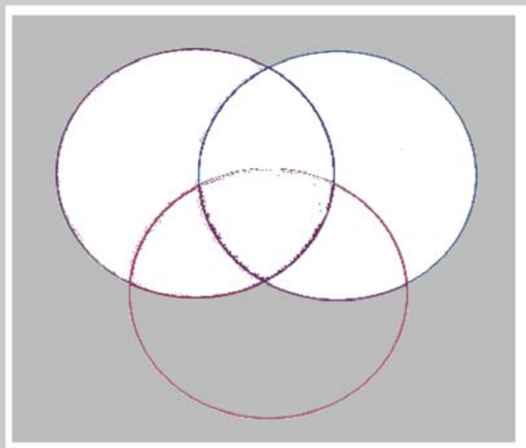
- By looking at the Venn diagram we can clearly see that we have 4 who do Spanish only.

Answer (c)

- By shading $E \cup F$ we would have:



- Therefore $(E \cup F)'$ will be the opposite of this shading.



Logic and truth tables

Propositions – A statement that can be either true or false. For example, the baby is a girl.

Negation – A proposition that has become negative: For example, the baby is not Australian. Be careful that it is a negative and not the opposite. The symbol used is $\neg$.

p	$\neg p$
T	F
F	T

Conjunction – the word 'and' used to join two conjunctions together. For example, the baby is a girl and Australian. The symbol used is $\wedge$.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction – the word 'or' used to join the conjunctions together. For example, the baby is a girl or Australian. The symbol used is $\vee$.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Implication – Using the words *if …… then* with two propositions. For example: If you do not sleep tonight then you will be tired tomorrow. The symbol used for implication is $\Rightarrow$.

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

By using the implication it can generate converse, inverse, and contrapositive. Basically these are:

$p \Rightarrow q$ – the implication
$q \Rightarrow p$ – the converse
$\neg p \Rightarrow \neg q$ – the inverse
$\neg q \Rightarrow \neg p$ – the contrapositive

The truth table can be constructed to give any one of these converse, inverse and contrapositive by using negation and the implication tables. However, you will need to know how to describe these in words.

Equivalence – Using the words 'if and only if' or 'is necessary and sufficient'. This applies to two statements that have the same truth table, as the two statements have are known to be equivalent. That is they imply the same meaning. The symbol used is $\Leftrightarrow$.

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Tautology – a statement that produces True (T) throughout the column of the truth table.

Contradiction – a statement that produces False (F) throughout the column of a truth table.

Validity of an argument – an argument is only valid if the compound proposition is considered to be a tautology, that is the column will be made up of T's.

Guided example

Let the propositions p, q, and r be defined as:

p: Andrea studies English at IB.
q: Andrea studies Spanish at IB.
r: The school offers at least 2 languages at IB.

(a) Write the following in logical form. If Andrea studies English and Spanish then the school offers at least 2 languages.
(b) Write the following statement in words:
$\neg p \Rightarrow \neg q$

(c) Copy and complete the truth table below.

p	q	r	$(p \wedge q)$	$\neg(p \wedge q)$	$\neg(p \wedge q) \Rightarrow r$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

(d) Explain the significance of the final statement: $\neg(p \wedge q) \Rightarrow r$.

Answer (a)

- The words if … then are in the statement so it must be: $(p \wedge q) \Rightarrow r$

Answer (b)

- The statement reads 'If not p the not q'
- By putting in words we will get *If Andrea does not study English then she will not study Spanish*.

Answer (c)

- Although these truth tables can appear daunting at first, taking each column one at a time makes things quite simple. $(p \wedge q)$ only gives a T if both p and q are T.
- $\neg(p \wedge q)$ is the opposite of $(p \wedge q)$. All the T turns to F, and vice versa.
- The final statement is the if … then. Remember to put the $\neg(p \wedge q)$ first and that your final column will only be F if you start have T F.
- The final truth table will be:

p	q	r	$(p \wedge q)$	$\neg(p \wedge q)$	$\neg(p \wedge q) \Rightarrow r$
T	T	T	T	F	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	F	T	T
F	T	F	F	T	F
F	F	T	F	T	T
F	F	F	F	T	F

Answer (d)

- This question is asking you whether or not the statement is logically valid. Since the compound proposition $(\neg(p \wedge q) \Rightarrow r)$ has a mixture of T's and F's in it's final column we do not have a tautology and the argument is not logically valid.